

On the motion of fluid in a moving domain, applications to fluid structure

Yadong Liu¹, Martin Kalousek², Sourav Mitra³, Šárka Nečasová⁴

SUMMARY

Problems of fluid flow inside a moving domain deserve a lot of interest as they appear in many practical applications. Such problems can also be seen as a preparation step for research of fluid-structure interaction problems. Research of the compressible version of the Navier-Stokes system dates back to the nineties when the groundbreaking result of the existence of the global weak solutions to the compressible barotropic Navier–Stokes system on a fixed domain was proved by P. L. Lions [4] and, later, by E. Feireisl and collaborators [1] who extended the existence result to more physically relevant state equations. After that the theory of weak solutions was extended to the problem of fluid flow inside a moving domain by Kreml et al. [2]. Such existing theory was applied to more complicated problem e.g. into the case of two compressible mutually noninteracting fluids and a shell of Koiter type encompassing a time dependent 3D domain filled by the fluids, [3]. Further, we will also consider just monofluid system and the elastic structure is part of the moving boundary of the fluid, and the Navier-slip type boundary condition is taken into account. Depending on the reference geometry (flat or not), we show the existence of weak solutions to the coupled system provided the adiabatic exponent satisfies $\gamma > \frac{12}{7}$ without damping and $\gamma > \frac{3}{2}$ with structure damping, utilizing the domain extension and regularization approximation. Moreover, via a modified relative entropy method in time-dependent domains, we prove the weak-strong uniqueness property of weak solutions. Finally, we give a rigorous justification of the incompressible inviscid limit of the compressible fluid-structure interaction problem with a flat reference geometry, in the regime of low Mach number, high Reynolds number, and well-prepared initial data, [5].

Keywords: compressible fluids, weak solutions, time-dependent domains, fluid-structure

AMS Classification: 76N, 35Q30

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¹School of Mathematical Sciences,
Nanjing Normal University,
Nanjing 210023, P. R. China email: ydliu@njnu.edu.cn

²Institute of Mathematics,
Czech Academy of Sciences,
Žitná 25, 115 67 Praha, Czech Republic email: kalousek@math.cas.cz

³Department of Mathematics
Simrol, Indore, 453552,
Madhya Pradesh, India email: souravmitra@iiti.ac.in

⁴Institute of Mathematics,
Czech Academy of Sciences,
Žitná 25, 115 67 Praha, Czech Republic email: matus@math.cas.cz